

Evaluation of Continuous Sampling Plan Indexed through Minimum Angle and Maximum Acceptance Quality

¹V. Nirmala, ²K.K. Suresh

¹Women Scientist - A, Department of Science and Technology, Govt. of India (DST-WOS-A),
Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India

²Professor and Head, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu,
India

E-mail: ¹vnirmalakanna@yahoo.com, ²sureshkk1@rediffmail.com

Abstract

In this paper, an evaluation and quality designing methodology is designed to determine the quality measures in a new procedure for Continuous Sampling Plan –M indexed through Minimum Angle method and Maximum Acceptance Quality. Tables and procedures are provided for the selection of the parameter for the plan. Numerical designs are also provided for the shop floor applications of the manufacturing industries. Minimum Angle indexed plan provides a method for designing sampling plan based on higher quality product selection with minimum inspection cost and time, instead of classical determination about quality in operating characteristic (OC) curve measurement. Minimum Angle method is to provide wider potential applicability in manufacturing industry ensuring higher standard of quality product selection attainment.

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INTRODUCTION

Acceptance sampling is a quality scheme for the customer to not accept bad lots as well as the manufacturer to accelerate the process control. In an advanced improvement of production with increased chances of event of non-conforming items, Statistical process control enhances the process capability and acceptance sampling performances logically to preclude passing out non-conforming units.

The paper introduces a method for Minimum Angle. This method seems to be versatile and can be adopted in the elementary production process where the stipulated quality level is advisable to fix at later stage and provides a new concept for Selection of Continuous Sampling Plan-M.

Review on continuous sampling plan of

type CSP-M

Multilevel continuous sampling plan stretch back to the development work of Lieberman and Solomon (1955) generating a whole series of papers and reports on the new procedures. The designation CSP-M is from MIL-STD-1235, but the plan is carried over from Handbook H-106 (1958). CSP-M differs from the original multilevel continuous sampling plans principally in the incorporation of the rule of four from CSP-3 as a protection against spotty quality that may otherwise be difficult to detect as higher sampling levels (being powers of the original fraction f) result in infrequent or reduced sampling. Its basic parameters are i , f and k , the latter being the maximum number of levels of reduction in the sampling fraction, specifically, the maximum power of ' f ' employed as a sampling fraction. Based on good quality performance, as measured by the sampling procedure itself, reductions

in sampling from f to $f_2, \dots, f_k, \dots, f_k$ are permitted. Where $k=1, 2, \dots, k$ and with 'k' varying between 1 and 5 in increments of 1 for asecific plans. For $k=1$, CSP-M becomes CSP-3.

CSP-M requires that the inspection rate be increased (lower power of f) whenever a defective unit is found during sampling inspection and either one of the next four units to pass the inspection station is defective or one of the next $i-4$ units inspected is defective. The increase in inspection rate is not required if a defective unit is found during sampling inspection provided that all of the next $i-4$ units sampled and inspected (including the 4 consecutive units to follow the defective unit to the inspection station) are found to be defect-free. On the other hand, a reduction in inspection rate (higher power of f to maximum of k) is permitted if the first i units sampled and inspected at the frequency f_k are defect-free ($k=1, 2, \dots, k-1$) or if a defective unit is found while sampling at the frequency f_k and the next 4 consecutive units are defect-free and the next $i-4$ sampled and inspected units are defect-free ($k=1, 2, \dots, k-1$).

Operating Procedure of CSP-M plan

1. Verification sampling during the 100% screening phase with the requirement that sampling inspection shall not be instituted if any of the i consecutive units found defect-free by the screening crew are found defective by the verification inspector(s).
2. Use of the stopping rule during the 100% screening phase with the requirement, from the tables "if during a period of 100% inspection the number of units inspected exceeds the appropriate value. The consumer may, at its option, suspend acceptance until the producer corrects the cause of the high rate of defectives. And after the corrective action is taken, 100% inspection may be resumed".

Provision for tightened and reduced sampling is an integral part of the multilevel sampling procedure. Only the AOQ curve for the CSP-M plans is given in the referenced standard.

Selection Procedure for Continuous Sampling Plan of type CSP-M using Minimum Angle Method:

The practical performance of any sampling plan is revealed through its Operating Characteristic curve. In order to satisfy the consumer's requirement and producer's profit, it is very important to minimize the both consumer and producer risks. Both the risks can be minimized when the ideal OC curve is made to pass closely through $(AOQ, 1-\alpha)$ and (AOQ, β) for any designed sampling plan.

Peach and Littauer (1946) have studied and analysed two points of OC curve as $(p_1, 1-\alpha)$ and (p_2, β) for the ideal OC curve situation to minimize the consumer risk. Norman Bush et.al (1953) considered two points on the OC curve as $(AOQ, 1-\alpha)$ and (LOL, β) towards minimizing the risks by considering tangent angle of the point of inflection. With this idea another approach to minimize the risks for ideal conditions is to minimize the tangent angle θ between the lines joining the points $(AOQ, 1-\alpha)$, (AOQ, β) and $(AOQ, 1-\alpha)$, (LOL, β) was proposed by Singaravelu (1993). Applying this method one can get a better plan which has an OC curve approaching to the ideal OC curve. Thus, the value of performance criterion that a tangent angle θ made by the above mentioned points is given as,

$$\tan \theta = \frac{p_2 - p_1}{P_a(p_1) - P_a(p_2)}$$

Where $p_1 = AOQ$ and $p_2 = LQL$. Thus the value of angle θ can be calculated by,
 $\theta = \tan^{-1}(\tan(\theta))$

Operating Procedure and performance measures of CSP-M plan

Multilevel continuous sampling plan stretch back to the development work of Lieberman and Solomon (1955) generating a whole series of papers and reports on the new procedures.

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Provision for tightened and reduced sampling is an integral part of the multilevel sampling procedure. Only the AOQ curve for the CSP-M plans is given in the referenced standard.

Construction of Tables

Table: 1. Parametric values of Continuous Sampling Plan CSP-M with Minimum angle method for $f=1/2$

| i | p_1 | $P_a(p_1)$ | p_2 | $P_a(p_2)$ | OR | $\tan \theta$ | Angle (θ) |
|-----|----------|------------|----------|------------|----------|---------------|--------------------|
| 100 | 0.006453 | 0.95 | 0.059159 | 0.1 | 10.88887 | 0.064745 | 4.654303 |
| 150 | 0.006641 | 0.95 | 0.042509 | 0.1 | 5.815185 | 0.041411 | 3.372652 |
| 200 | 0.009365 | 0.95 | 0.03343 | 0.1 | 4.573187 | 0.030729 | 2.760666 |
| 250 | 0.006768 | 0.95 | 0.027704 | 0.1 | 4.047925 | 0.024541 | 2.406106 |
| 300 | 0.006379 | 0.95 | 0.023742 | 0.1 | 3.7219 | 0.020427 | 1.170384 |
| 350 | 0.006003 | 0.95 | 0.020815 | 0.1 | 3.467433 | 0.017426 | 1.998437 |
| 400 | 0.005724 | 0.95 | 0.018561 | 0.1 | 3.30032 | 0.01522 | 0.992042 |
| 450 | 0.005242 | 0.95 | 0.016773 | 0.1 | 3.199733 | 0.013566 | 0.977268 |
| 500 | 0.004964 | 0.95 | 0.015299 | 0.1 | 3.08199 | 0.012159 | 0.696649 |
| 550 | 0.006720 | 0.95 | 0.01409 | 0.1 | 2.991507 | 0.011035 | 0.832276 |
| 600 | 0.004654 | 0.95 | 0.013087 | 0.1 | 2.938258 | 0.010156 | 0.781923 |
| 650 | 0.004870 | 0.95 | 0.012205 | 0.1 | 2.863679 | 0.009345 | 0.635412 |
| 700 | 0.004153 | 0.95 | 0.011441 | 0.1 | 2.822847 | 0.008692 | 0.598001 |
| 750 | 0.003453 | 0.95 | 0.010777 | 0.1 | 2.769013 | 0.0081 | 0.464096 |
| 800 | 0.003672 | 0.95 | 0.010189 | 0.1 | 2.741189 | 0.007614 | 0.396257 |

Table: 2. Parametric values of Continuous Sampling Plan CSP-M with Minimum angle method for $f=2/3$

| i | p_1 | $P_a(p_1)$ | p_2 | $P_a(p_2)$ | OR | $\tan \theta$ | Angle (θ) |
|-----|----------|------------|----------|------------|----------|---------------|--------------------|
| 100 | 0.007825 | 0.95 | 0.060955 | 0.1 | 7.891615 | 0.062624 | 4.589064 |
| 150 | 0.008693 | 0.95 | 0.04367 | 0.1 | 5.023582 | 0.041149 | 4.357688 |
| 200 | 0.008273 | 0.95 | 0.034313 | 0.1 | 4.147589 | 0.030635 | 3.755273 |
| 250 | 0.007685 | 0.95 | 0.028404 | 0.1 | 3.696031 | 0.024375 | 3.396601 |
| 300 | 0.007095 | 0.95 | 0.024334 | 0.1 | 3.429739 | 0.020281 | 2.162026 |
| 350 | 0.006536 | 0.95 | 0.021331 | 0.1 | 3.263617 | 0.017406 | 1.997284 |
| 400 | 0.006123 | 0.95 | 0.018995 | 0.1 | 3.102237 | 0.015144 | 1.86766 |
| 450 | 0.005735 | 0.95 | 0.017151 | 0.1 | 2.990584 | 0.013431 | 0.976951 |
| 500 | 0.00538 | 0.95 | 0.015676 | 0.1 | 2.913755 | 0.012113 | 0.969402 |
| 550 | 0.005072 | 0.95 | 0.01443 | 0.1 | 2.845032 | 0.011009 | 0.830793 |
| 600 | 0.004783 | 0.95 | 0.013366 | 0.1 | 2.79448 | 0.010098 | 0.785853 |
| 650 | 0.004556 | 0.95 | 0.012482 | 0.1 | 2.739684 | 0.009325 | 0.634266 |

| | | | | | | | |
|-----|----------|------|----------|-----|----------|----------|----------|
| 700 | 0.004333 | 0.95 | 0.011699 | 0.1 | 2.699977 | 0.008666 | 0.596518 |
| 750 | 0.004153 | 0.95 | 0.011015 | 0.1 | 2.6523 | 0.008073 | 0.462545 |
| 800 | 0.003974 | 0.95 | 0.010412 | 0.1 | 2.62003 | 0.007574 | 0.339654 |

Table: 3. Parametric values of Continuous Sampling Plan CSP-M with Minimum angle method for $i=400$

| f | p_1 | $P_a(p_1)$ | p_2 | $P_a(p_2)$ | OR | $\tan \theta$ | Angle (θ) |
|-----|----------|------------|----------|------------|----------|---------------|--------------------|
| 1/2 | 0.005646 | 0.95 | 0.018761 | 0.1 | 3.30132 | 0.01522 | 0.987342 |
| 1/3 | 0.00421 | 0.95 | 0.017305 | 0.1 | 4.210462 | 0.015524 | 0.989465 |
| 1/5 | 0.002904 | 0.95 | 0.016308 | 0.1 | 5.615702 | 0.015769 | 0.903521 |
| 1/7 | 0.001704 | 0.95 | 0.015411 | 0.1 | 9.044014 | 0.016126 | 0.823945 |
| 2/3 | 0.006123 | 0.95 | 0.018995 | 0.1 | 3.102237 | 0.015144 | 0.806766 |
| 2/5 | 0.00499 | 0.95 | 0.017991 | 0.1 | 3.605411 | 0.015295 | 0.763586 |
| 2/7 | 0.003919 | 0.95 | 0.017135 | 0.1 | 4.372289 | 0.015548 | 0.690848 |
| 3/5 | 0.006123 | 0.95 | 0.018995 | 0.1 | 3.102237 | 0.015144 | 0.56766 |
| 3/7 | 0.00499 | 0.95 | 0.017991 | 0.1 | 3.605411 | 0.015295 | 0.476356 |
| 4/7 | 0.005986 | 0.95 | 0.01888 | 0.1 | 3.154026 | 0.015169 | 0.369143 |
| 4/9 | 0.005243 | 0.95 | 0.018236 | 0.1 | 3.478161 | 0.015286 | 0.275817 |
| 5/9 | 0.00589 | 0.95 | 0.018796 | 0.1 | 3.191171 | 0.015184 | 0.169956 |

The expression for $P_a(p)$ of Continuous Sampling Plan of type CSP-M under the condition $k=i$ is given by,

$$P_a(p) = \frac{q^i(f^i + fi)}{f^{i+1} + q^i(f^i + fi - f^{i+1})}$$

Where $q=1-p$ and p is the incoming proportion defective and provides the probability of acceptance for the submitted lot.

To obtain the optimal sampling plan for the wide range of p_1 and p_2 with the specified conditions of producer and consumer risks we follow the following steps.

1. Find p_1 and p_2 such that $P_a(p_1) \geq 1 - \alpha$ and $P_a(p_2) \leq \beta$.
2. Compute the Operating Ratio (OR) = p_2/p_1 .
3. Calculate the $\tan \theta$ value using the equation 4.1.1 and calculate the angle θ by equation 4.1.2.
4. Record the values of angle θ , among the values of θ , a minimum value is chosen and let it be θ_m .
5. Thus the optimum plan is determined with minimum angle θ_m .

To determine the optimal plan CSP-M, we employ the procedure of minimum angle

method as mentioned above for a wide range p_1 and p_2 with specified risks of producer and consumer. A set of tables are tabulated with maximum producer's risk of 5% and maximum consumer's risk of 10% for different range of parameters.

Illustration

Suppose the strength of the plan is specified as $f=1/2$, $\alpha=0.05$ and $\beta=0.10$ then it can be seen from the Table 1 for different values of clearance number i we get θ . Thus, among all the values of angle θ we consider a minimum angle found as $i=250$ and $f=1/2$ which will satisfies both consumer and producer with low proportion defectives.

It is also observed from the Tables 1, 2 that for fixed values of f , while the clearance number i increases, the operating ratio increases and the tangent angle decreases at all levels of p . All the tables ensure us the protection to consumer against bad quality. It is clear from these tables that producer's interest will be safeguarded when smaller clearance numbers are used.

Using the minimum angle method procedure as mentioned above, the optimal sampling plans are determined for a fixed f

for the wide range p_1 and p_2 with specified producer's and consumer's risks and are tabulated in table 1 and 2. The plans obtained from Tables 1, 2 and 3 will have a maximum producer's risk of 5% and a maximum consumer's risks of 10%.

Thus by employing angle method one can obtain the optimal sampling plan for any fixed parameters with low proportion defective which will protect the consumer against bad quality and safeguard producer with smaller clearance numbers.

CONCLUSION

Bayesian Acceptance sampling is the advanced procedure, which contracts with the measures in which pronouncement to accept or reject lots or process based on their inspection of past history or information of samples. The present work mainly relates to the Construction and Selection of performance measures for Bayesian Conditional Repetitive Group Sampling Plan indexed through Quality regions. Tables are provided which are tailor-made, handy and ready-made uses to their industrial shop-floor situations.

Quality Interval Sampling plan possesses broader probable applicability in manufacturing industry guaranteeing higher standard of quality achievement for any product or process. This is ready-made use to manufacturing industrial shop-floor conditions. The present improvement would be valuable calculation to the literature and a beneficial device for quality control practitioners and quality engineers.

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